A Mathematical Version of Garrison's Model

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ABSTRACT: We develop a simple mathematical version of Garrison's model. The purpose to develop a mathematical framework is to (1) show how such representation can be used and (2) layout a path for future work that requires a more flexible version of Garrison's treatment than the graphical exposition. While the graphical model is limited to three dimensions, a mathematical version can include more variables of interest. First, we develop the mathematical framework of Garrison's treatment. Then we apply it to the cases of increase in savings, secular growth, and the Austrian business cycle theory.

KEYWORDS: business cycle, Austrian School, Garrison

JEL CLASSIFICATION: B53, E32

1. INTRODUCTION

The contemporary literature on the Austrian business cycle theory (ABCT) is notably influenced by Garrison's model (2001).

This model offers clear guidelines to highlight the distinctive aspects embedded in the ABCT, specifically the effects of interest rate movements in the structure of production. The impact of Garrison's model has been of such extent that, sometimes, it seems that Garrison's model is being interpreted as being *the* ABCT instead of being *one of the possible* representations of the ABCT.[1]

In the theoretical literature, different extensions to the model have tried to account for open economies, growth, and risk (Cachanosky, 2014b; Fillieule, 2005; Ravier, 2011; Young, 2009, 2015). These papers extend Garrison's models work by adding missing pieces that would allow for the model to offer a better explanation to business cycles such as the subprime crisis. In the empirical literature, the model has been used to illustrate how the predictions of the model can be seen with the available data. Typically, data at the industrial level are categorized as different stages of production and then the observed behavior is compared with the model's predicted behavior (Lester and Wolff, 2013; Luther and Cohen, 2014; Mulligan, 2002, 2013; Powell, 2002; Young, 2005, 2012, 2015).[2] Both of these approaches present challenges. The literature shows that extensions to the model are not easy to display or interpret and that the empirical work requires putting forward assumptions too resrictive to either be realistic or offer valuable results.

Furthermore, according to Garrison (2001, p. xii), the graphical representation he offers should be interpreted to be more a pedagogical tool than a model to drive empirical reseach and develop theoretical nuances of the ABCT (*italics* original, **bold** added):

> In the early 1970s I entered the graduate program at the University of Missouri, Kansas City, where I learned the intermediate and (at the time)

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[1] While we are not arguing this is a "bad" thing, the model and Hayek's triangle have also received some critical reviews (Barnett II & Block, 2006; Hülsmann, 2001). For an alternative framework to the ABCT in the field of finance, see Cachanosky & Lewin (2016) and Lewin & Cachanosky (2016).

[2] Some authors offer an alternative approach; instead of categorizing industries as stages of production, the interest rate sensitivity of industries is compared. In the Garrison's model framework, this means that each industry is argued to have a Hayekian triangle of a different size regardless of its position as a stage of production in the production structure (Cachanosky, 2014a, 2015b; Young, 2012). This approach does not deal with the problem of defining stages of production and still looks at industrial level data rather than aggregates.

advanced versions of Keynesianism. Having read and by then reread the *General Theory*, **the ISLM framework struck me as a clever pedagogical tool** but one that, like Samuelson's gloss, left the heart and soul out of Keynes's vision of the macroeconomy. **It was at that time that I first conceived of an Austrian counterpart to ISLM** – with a treatment of the fundamental issues of the economy's self-regulating capabilities emerging from a comparison of the two contrasting graphical frameworks.[3]

Garrison's model value is also one of its main limitations. Like a demand and supply graph, Garrison's model is able to say a great deal with just a few lines. But because Garrison's model is a graphical one, it can only deal with at most three relationships (dimensions) at once. Besides the rapid increase in graphical complexity, the model is limited in the number of relationships it can handle at the same time. It is noteworthy that given the influence of Garrison's model on contemporary ABCT literature, there is no mathematical framework of Garrison's model that would allow for a more flexible model. If a graphical model exists, then a mathematical version is already implied in the model. This is the contribution of this paper. We introduce a mathematical, and arguably simple, model of Garrison's graphical model. This simple model is not intended to be a definite version of Garrison's model not to change what the model has to say, but a first step toward more complex and flexible versions as the contemporary applied ABCT literature seems to require.

The next section develops the mathematical model for Garrison's model. Section 3 applies the model to two scenarios, increase in savings and secular growth. Section 4 applies the model to the ABCT case. Section 5 offers some suggestions of how this framework can be extended to offer different variations on a theme. Section 6 offers concluding remarks.

## 2. A MATHEMATICAL MODEL FOR GARRISON'S MODEL

Our mathematical version of Garrison's model requires making a few simplifications. The main difference between our version and

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[3] For Garrison (2001, p. xiii) the model goes from being a pedagogical tool to be an instrument of persuasion (in the classroom): "But because the interlocking graphics impose a certain discipline on the theorizing, they help in demonstrating the coherence of the Austrian vision. For many students, then, the framework goes beyond exposition to persuasion."

Garrison's model is that we use a linear production possibilities frontier (PPF). The reason for this is that a model with linear PPF facilitates algebraic calculations. As stated in the introduction, the purpose of this model is to offer some mathematical foundations to Garrison's model, not a complex or a more realistic model. Figure 1 depicts the Garrison's model we use in this paper.[4]

Figure 1: Garrison's Model with a Linear PPF
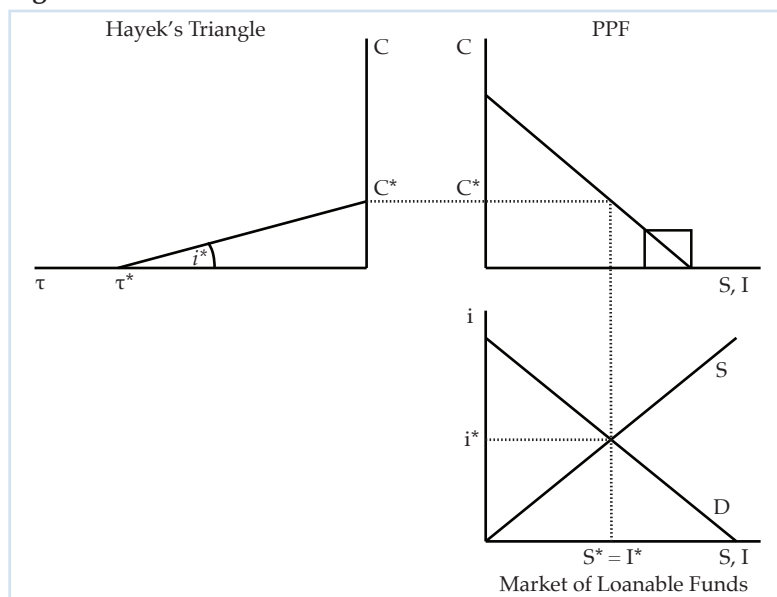
Before presenting the mathematical counterpart of this graph, a few clarifications are required. First, the interest rate defined in the market of loanable funds should be interpreted as a rate that *represents* the market yield (interest) curve. Investment decisions are valued with long-term interest rates, not with short-term interest rates (i.e. federal funds rate.) The ABCT argues that a credit expansion by the Federal Reserve puts into motion ABCT effects *if* the discount rate used by investors is affected as well. Put differently, this representation implicitly assumes parallel shifts of

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[4] This would be figure 3.7 in Garrison (2001, p. 50).

the yield curve, but no changes in the slope of the yield curve.[5] Second, the PPF is not represented in terms of units of goods, like guns and butter, but in dollar amounts. This also means that one more dollar spent in consumption (investment) is one less dollar spent in investment (consumption) making a straight line PPF with slope negative one a plausible assumption. Total income (Y) is separated into consumption (C) and investments (I) (that in equilibrium is equal to savings [S]). This means that monetary illusion can confuse nominal increases of C and I with real increases (the exact location of the PPF is uncertain). Third, the base of the Hayekian triangle is intended to capture Böhm-Bawerk's average period of production (APP). This means that the base of the triangle does not measure pure-time, but value-time. As Garrison (2001, p. 49) clarifies, "[t]wo dollars' worth of resources tied up in the production process for three years amounts to six dollar-years (neglecting compounding) of production time." Because the triangle assumes a constant flow of value-time, the APP is located in the middle of the base of the triangle. The length of the base (τ), then, measures the total period of production (TPP). The fact that the APP is one half of the TPP rests on a set of important assumptions. First, there is no compounding of returns. Second, there is a constant flow of value-in-time (this explains why the triangle hypotenuse is a straight line).[6] Finally, Austrians usually object to the interpretation that, in the ABCT, there is *over*investment when the theory argues for *mal*investment. The model, however, is open to such confusion. The PPF is in aggregate terms and Garrison shows how the economy locates itself (temporarily) beyond its potential output where the level of investment is *above its potential* or when the unemployment is below its natural rate. τ increases as well. This suggests *over*investment. *More* roundabout methods of production can also be interpreted as *over*investment rather than *mal*investment because this concept is associated with capital intensity. We do not claim that the ABCT argues for *mal*investment while Garrison's model argues that the main problem is

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[5] Bernanke and Blinder (1992, 919) argue that the federal funds rate "is a good indicator of monetary policy," and that the "Federal fund rate is *particularly* informative [of future movements in real macroeconomic variables]."

[6] For a more detailed discussion, see Cachanosky and Lewin (2014a), Cachanosky and Lewin (2014b) and Lewin and Cachanosky (2014).

*over*investment, but it should be pointed out that the model itself is open to the latter interpretation.

The model has four equations, (1) supply and (2) demand for loanable funds, (3) the PPF, and (4) Hayek's triangle hypotenuse. The unknowns in the model are $I, r, C,$ and τ.

(1) $I^D = A - \alpha i$

(2) $I^S = B + \beta i$

(3) $\bar{Y} = C + I$

(4) $C = i\tau$

Where I^D and I^S are the demand (investment) and supply (savings) for loanable funds respectively. $\bar{Y}$ is a given value of total output that is divided between consumption (C) and investment (I); this is the PPF. We should note that we assume this is a closed economy with no government.[7] The Hayekian triangle's hypotenuse is represented by the fourth equation, which has a zero intercept and slope i. Also $A, B > 0, A > B$, and $\alpha, \beta > 0$.

The model can easily be solved. First, from the market of loanable funds we can obtain the interest rate and investment values of equilibrium. Second, the equilibrium level of investment can be used to obtain the equilibrium level of consumption. Third, with the level of consumption and of the interest rate the total and average period of production in equilibrium can be calculated.

(5) $i^* = \dfrac{A - B}{\alpha + \beta}$

(6) $I^* = \dfrac{\beta A + \alpha B}{\alpha + \beta}$

(7) $C^* = \bar{Y} - \dfrac{\beta A + \alpha B}{\alpha + \beta} = \dfrac{(\alpha + \beta)\bar{Y} - (\beta A + \alpha B)}{\alpha + \beta}$

(8) $\tau^* = \dfrac{(\alpha + \beta)\bar{Y} - (\beta A + \alpha B)}{A - B}$

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[7] For a treatment of Garrison's model with government, see Ravier and Cachanosky (2015).

$$(9)\ APP^* = \frac{1}{2} \cdot \tau^*$$

An increase in the demand for loanable funds ($\Delta A > 0$) or a reduction in the slope of the demand ($\Delta\alpha < 0$) implies an increase in $i^*$and I^*. Similar effects can be tracked for changes in the supply of savings in the market for loanable funds through a comparative static analysis of each parameter for i^* or I^*.

We should note that the consumption function is a linear function with an intercept $\bar{Y}$ and a slope equal to negative one with respect to I^*. This also means that, in our model, all else equal, an increase in $\bar{Y}$ results in an increase in consumption but not in investment. This is because the PPF is assumed to be linear where each dollar that is not spent in C is spent in I. An increase in demand ($\Delta A > 0$) or supply ($\Delta B > 0$) for loanable funds reduces the level of consumption as more resources are devoted to investment *given* a level of output. Finally, we can obtain τ (TPP) and the APP from the Hayekian triangle. The total and average periods of production are directly related to the size of the economy ($\bar{Y}$). Since τ^* has to be positive, it follows from equations (7) and (9) that investment cannot be larger than the output: $\bar{Y} \geq \dfrac{\beta A + \alpha B}{\alpha + \beta} = I.$

We can calculate the area of the Hayekian triangle (H) which is the sum of all stages of production. This would be analogous to the gross domestic expenditures (GDE).[8] This area amounts to the total time-value investment of the structure of production and can be obtained by multiplying t with C and dividing by two:

$$(10)\ H = \frac{1}{2} \cdot \frac{[(\alpha+\beta)\bar{Y} - (\beta A + \alpha B)]^2}{(\alpha+\beta)(A-B)}$$

3. APPLICATIONS

3.1 Increase in Savings

A change in time preference towards an increase in savings can be captured by a positive change in B ($\Delta B > 0$). This means

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[8] The Gross Domestic Product (GDP) equals Gross Output (GO) plus Intermediate Expenditures (IE), and GO equals GDP plus Intermediate Investment (II). Then, GO = GDP + II and GDE = GO + IE.

that, at the same interest rate in the market, economic agents are willing to supply more loanable funds. The comparative statics are straightforward.

(11) $\dfrac{\partial i^*}{\partial B} = -\dfrac{1}{\alpha+\beta} < 0$

(12) $\dfrac{\partial I^*}{\partial B} = \dfrac{\alpha}{\alpha+\beta} > 0$

(13) $\dfrac{\partial C^*}{\partial B} = -\dfrac{\alpha}{\alpha+\beta} < 0$

(14) $\dfrac{\partial \tau^*}{\partial B} = \dfrac{(\alpha+\beta)(\bar{Y}-A)}{(A-B)^2} \lesseqgtr 0$

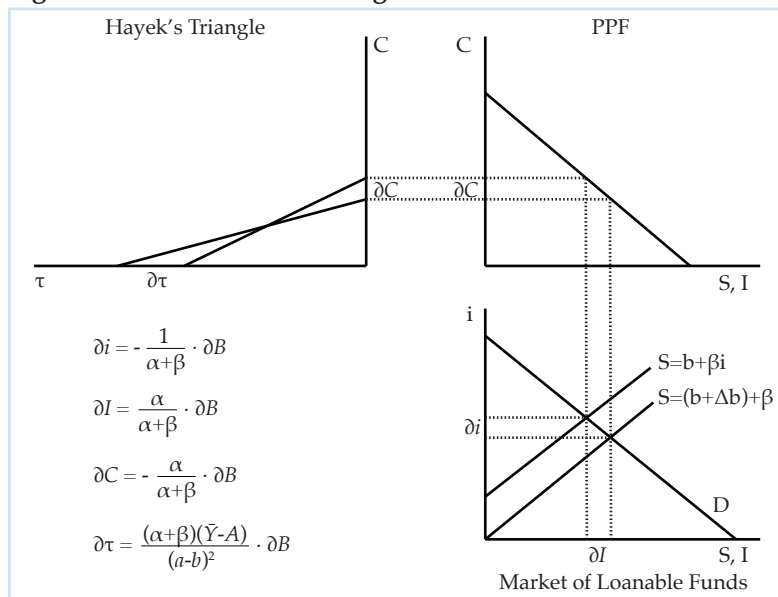As expected, the increase in savings reduces the interest rates. It results also in an increase in investment equal to the reduction in consumption $\left(\dfrac{\partial I^*}{\partial B} + \dfrac{\partial C^*}{\partial B} = 0 \right)$. But the effect on τ (and, therefore, on the APP) depends on the sign of $(\bar{Y}\text{-}A)$. Intuitively, this captures the opposite effects on APP of (1) a fall in interest rates and (2) a fall in consumption. Finally, we should add that, because, $\dfrac{\partial C^*}{\partial B}$, if $\dfrac{\partial \tau^*}{\partial B}$, then $\dfrac{\partial H^*}{\partial B}$ (the area of the Hayekian triangle decreases as well because both, height (C) and width (τ) are falling). Figure 2 shows the results (with an increase in τ).[9]

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[9] This would be Figure 4.2 in Garrison (2001, p. 62).

Figure 2: An Increase in Savings in Garrison's Model

Hayek's Triangle

PPF

$$\partial i = -\frac{1}{\alpha+\beta} \cdot \partial B$$

$$\partial I = \frac{\alpha}{\alpha+\beta} \cdot \partial B$$

$$\partial C = -\frac{\alpha}{\alpha+\beta} \cdot \partial B$$

$$\partial \tau = \frac{(\alpha+\beta)(\bar{Y}-A)}{(a-b)^2} \cdot \partial B$$

$S=b+\beta i$

$S=(b+\Delta b)+\beta$

D

Market of Loanable Funds

3.2 Secular Growth

Garrison (2001, Chapter 4) presents the case of secular (technology-induced) growth. Garrison assumes that the technology growth has no effect on the level of interest rates. This case can be divided in two steps. First, the new technology increases the demand for savings by the firms. Second, there is an increase in the supply of savings *after* income increases. Therefore, the interest rate rises first and then it returns to its original level. Figure 3 reproduces Garrison's (2001, p. 59) Figure 4.1.

Figure 3: Garrison's Secular (Technology-Induced) Growth
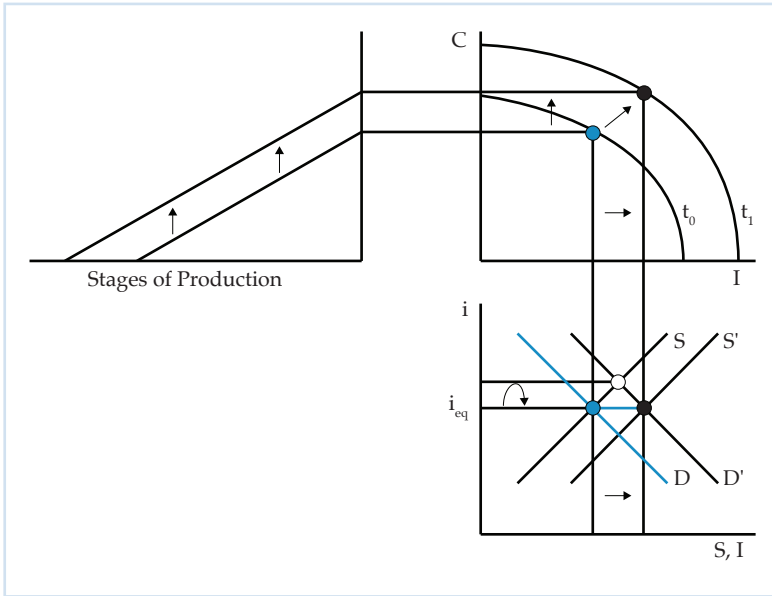
To follow Garrison's exposition as closely as possible, we need to make three modifications to our model. First, we modify the market for loanable funds to make demand and supply of savings depends on technology and income respectively; this allows following Garrison's two steps. Second, we need to add time (t). Third, we need to add a production function to capture growth. The model now becomes the following:

(15) $I_t^D = A_t(Z_t) - \alpha i_t$

(16) $I_t^S = B_t(Y_{t-1}) + \beta i_t$

(17) $Y_t = C_t + I_t$

(18) $C_t = i_t \tau_t$

(19) $Y_t = Z_t(K_t^{1-\gamma} \bar{L}_t^\gamma)$

(20) $K_t = (1-\delta) K_{t-1} + I_t$

Subscript t denotes time, Y is not a given value anymore and follows a Cobb-Douglas production function where Z is technology, K as capital, $\bar{L}$ as a given amount of labor, and $\gamma \in (0,1)$.

Finally, $\delta \in (0,1)$ is the depreciation rate. For a steady state where $K_{(t+1)} = K_t$, we need $I_t^* = \delta K_t$. This means that the equilibrium interest rate in the loanable funds market yields an investment value of δK_t. The equilibrium conditions now become the following:

$$(21)\ i^* = \frac{A_t(Z_t) - B_t(Y_{t-1})}{\alpha + \beta}$$

$$(22)\ I^* = \frac{\beta A_t(Z_t) + \alpha B_t(Y_{t-1})}{\alpha + \beta}$$

$$(23)\ C^* = \frac{(\alpha + \beta)\bar{Y} - (\beta A_t(Z_t) + \alpha B_t(Y_{t-1}))}{\alpha + \beta}$$

$$(24)\ \tau^* = \frac{(\alpha + \beta)Y_t - (\beta A_t(Z_t) + \alpha B_t(Y_{t-1}))}{A_t(Z_t) - B_t(Y_{t-1})}$$

$$(25)\ APP^* = \frac{1}{2} \cdot \tau(Z_t, Y_{t-1})^*$$

$$(26)\ K_t^* = \frac{\beta A_t(Z_t) + \alpha B_t(Y_{t-1})}{(\alpha + \beta)}$$

$$(27)\ Y_t^* = Z_t \left(\left(\frac{\beta A_t(Z_t) + \alpha B_t(Y_{t-1})}{\delta(\alpha + \beta)} \right)^{1-} \cdot \bar{L}_t \right)$$

3.2.1 Short-run effect

Taking this steady state as our initial position, assume now a positive shock to technology in period t.

$$(28)\ \frac{\partial i_t^*}{\partial Z_t} = \frac{A'(Z_t)}{\alpha + \beta} > 0$$

$$(29)\ \frac{\partial I_t^*}{\partial Z_t} = \frac{\beta A_t'(Z_t)}{\alpha + \beta} > 0$$

$$(30)\ \frac{\partial C_t^*}{\partial Z_t} = \frac{-\beta A_t'(Z_t)}{\alpha+\beta} > 0$$

$$(31)\ \frac{\partial \tau_t^*}{\partial Z_t} \lessgtr 0$$

$$(32)\ \frac{\partial K_t^*}{\partial Z_t} = \frac{\beta A_t'(Z_t)}{\delta(\ +\beta)} > 0$$

$$(33)\ \frac{\partial Y_t}{\partial Z_t} = 1 \cdot \left(\frac{\partial K_t(Z_t)^{1-\gamma}}{\partial Z_t}(Z_t) \cdot \bar{L}_t^{\gamma} \right) > 0$$

In the short run, the effect on τ depends on whether the increase in C (height of the triangle) more than compensates the increase in i (slope of the triangle); recall that $\tau_t = \dfrac{C_t}{i_t}$.[10] Note that output (equation 33) increases because there is better technology *and* because there is an increase in capital (equation 32). The excess of investment over capital depreciation increase income in future periods and, with this effect, there is an increase in the supply of savings.

3.2.2 Long-run effect

In period $t+1$ the investment and the stock of capital continue to increase. The increase in K continues until period $T \geq t+1$ where, again, $I_T^* = \delta K_T$.

$$(34)\ \frac{\partial K_{t+1}}{\partial Z_t} = \frac{\beta A_t(Z_t)}{(\alpha+\beta)} > 0$$

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10 $\dfrac{\partial \tau_t^*}{\partial Z_t} = \dfrac{\left[(\alpha+\beta)\dfrac{\partial Y_t}{\partial Z_t} - \beta A_t(Z_t)\right]\left[A_t(Z_t) - B_t(Y_{(t-1)})\right] - (\alpha+\beta)Y_t - \left[\beta A_t(Z_t) + \alpha B_t(Y_{(t-1)})\right]}{(A_t(Z_t) + B_t(Y_t - 1))^2} \lessgtr 0$
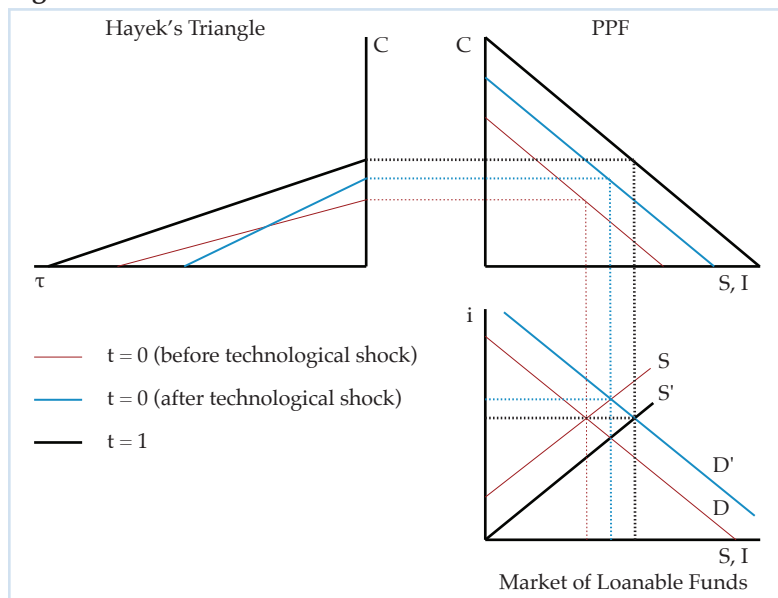
$$(35) \ \frac{\partial K_{t+1}}{\partial Y_t} = \frac{\alpha B_{t+1}'(Y_{t+1})}{(\alpha+\beta)} > 0$$

$$(36) \ \frac{\partial Y_{t+1}}{\partial K_{t+1}} = Z_{t+1} \left(\frac{L_{t+1}}{K_{t+1}} \right)^{\gamma} > 0$$

$$(37) \ \frac{\partial I_{t+1}^S}{\partial Y_t} = B_{t+1}'(Y_t) > 0$$

If the increase in I_T^S is such that $i_T^* = i_T^*$ then we obtain Garrison's secular growth graphical representation shown in Figure 3. The effects of our model are captured in Figure 4.

Figure 4: Garrison's Model with Secular Growth

4. GARRISON'S VERSION OF THE AUSTRIAN BUSINESS CYCLE THEORY

Garrison's representation of the ABCT overlaps Figure 1 with the effects of an expansion of credit by the monetary authorities. The monetary authorities' action results in a secondary supply of loanable funds that reduces i and produces an unstable situation where I and C try to increase at the same time beyond the limits of the PPF. The detachment of i from economic agents' time preference results in saving and investment not being equal anymore. The reduction in i increases τ, but the increase in consumption increases the height of the triangle. The inconsistency of trying to increase I and C (the boom) for a given $\bar{Y}$ pulls the triangle on both sides, "breaking" the hypotenuse of the Hayekian triangle. The exact location where the hypotenuse breaks depends on the slope and relative effects on C and τ. The longer this tension is in place and the farther away i is from the equilibrium level, the more *mal*investment is accumulated and the costlier the correction (the bust) will be. To capture Garrison's version of the ABCT we need to add a function that represents the supply of loanable funds with the monetary authority intervention (G).

(38) $I^D = A - \alpha i$

(39) $I^S = B + \beta i$

(40) $I^S_g = B + G + \beta i$

(41) $Y = C + I$

(42) $C = i\tau$

Where G represents the credit expansion by the monetary authorities. Garrison's model applied to the ABCT requires us to pay attention to three sets of points. First, the equilibrium values absent the central bank intervention, denoted with superscript * (already solved above). Second, the values that originate from the supply of credit with the monetary expansion of the central bank. These are denoted with a subscript g. Third, the values that originate from the supply of loanable funds without the government. These private market values are denoted with the subscript p. Following the same steps than above, we can solve the model for the case of credit expansion

$$(43)\ i_g^* = \frac{A-B-G}{\alpha+\beta}$$

$$(44)\ i_g^* = \frac{(\beta A+\alpha B)+\alpha G}{\alpha+\beta}$$

$$(45)\ C_g^* = \frac{(\alpha+\beta)\bar{Y}-(\beta A+\alpha B)-\alpha G}{\alpha+\beta}$$

$$(46)\ \tau_g^* = \frac{(\alpha+\beta)\bar{Y}-(\alpha B+\beta A)-\alpha G}{A-B-G}$$

$$(47)\ APP_g = \frac{1}{2}\left[\frac{(\alpha+\beta)\bar{Y}-(\alpha B+\beta A)-\alpha G}{A-B-G}\right]$$

From equation 46 we can calculate the change in τ when there is an increase in credit ($\Delta G>0$) and the elasticity of τ with respect to G $\left(\varepsilon_{\frac{\tau}{G}}\right)$. These two measures give us a proxy of the degree of roundaboutness sensitivity to the central bank intervention in the market for loanable funds.[11]

$$(48)\ \frac{\partial\tau}{\partial G} = \frac{(\alpha+\beta)(\bar{Y}-A)}{(A-B-G)^2}$$

$$(49)\ \varepsilon_{\frac{\tau}{g}} = \frac{(\alpha+\beta)(\bar{Y}-A)}{(\alpha+\beta)\bar{Y}-(\alpha B+\beta A)-\alpha G}\cdot\frac{G}{A-B-G}$$

We can also measure the deviations between the market position with the central bank intervening and the market position in the base case without the central bank intervening.

$$(50)\ i_g^* - i^* = -\frac{1}{\alpha+\beta}G$$

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[11] For the elasticity to be positive, the following two restrictions are required: (1)$((\alpha+\beta)\ \bar{Y}-(\beta A+\alpha B)-\alpha G)\cdot(A-B-G)>0$, (2)$(\bar{Y}-A)>0$.

$$(51)\ I_g^* - I^* = \frac{\alpha}{\alpha + \beta} G$$

$$(52)\ C_g^* - C^* = -\frac{\alpha}{\alpha + \beta} G$$

$$(53)\ \tau_g^* - \tau^* = \frac{(\alpha + \beta)(\bar{Y} - A) G}{(A - B)(A - B - G)}$$

We can now calculate the values for the market without the central bank intervening. In this case, the market reacts to i_g^* but yields an implicit i_p^* that represents the slope for late stages of production. This implicit rate is the one that prevails at the demand for loanable funds given the private supply of funds at i_g^*.

$$(54)\ I_p^* = \frac{(\beta A + \alpha B) - \beta G}{\alpha + \beta}$$

$$(55)\ i_p^* = \frac{(A - B)\alpha + \beta G}{\alpha(\alpha + \beta)}$$

$$(56)\ C_p^* = \frac{(\alpha + \beta)\bar{Y} - (\beta A + \alpha B) + \beta G}{\alpha + \beta}$$

$$(57)\ \tau_p^* = \frac{\alpha\left((\alpha + \beta)\bar{Y} - (\beta A + \alpha B) + \beta G\right)}{(A - B)\alpha + \beta G}$$

$$(58)\ APP_p = \frac{1}{2}\left[\alpha \frac{(\alpha + \beta)\bar{Y} - (\beta A + \alpha B) + \beta G}{\alpha(A - B) + \beta G}\right]$$

Similarly, we can measure the deviations of the market from the base scenario when the central bank intervenes in the market for loanable funds.

$$(59)\ I_p^* - I^* = -\frac{\beta}{\alpha + \beta} G$$

$$(60)\ i_p^* - i^* = \frac{\beta G}{\alpha(\alpha + \beta)}$$

$$(61)\ C_p^* - C^* = \frac{\beta}{\alpha+\beta}G$$

$$(62)\ \tau_p^* - \tau^* = \frac{(A-B)-(\alpha+\beta)\bar{Y}+(\beta A+\alpha B)}{(A-B)^2\alpha+(A-B)\beta G}$$

The credit expansion by the central bank pushes the economy beyond the PPF by the amount G, which is distributed between the deviation in investment and consumption.

$$(63)\ Y_g = \bar{Y} + G$$

$$(64)\ (I_g^* - I^*) + (C_p^* - C^*) = G$$

The next step is to calculate the difference between the economic variables affected by G and the market reaction to the central bank's monetary policy.

$$(65)\ i_g^* - i_p^* = -\frac{1}{\alpha}G$$

$$(66)\ I_g^* - I_p^* = G$$

$$(67)\ C_g^* - C_p^* = -G$$

$$(68)\ \tau_g^* - \tau_p^* = \frac{(\alpha+\beta)[(\alpha+\beta)Y-\alpha(A-B-G)]+(\beta A+\alpha B)-2\cdot\alpha\beta G}{(A-B-G)[\alpha(A-B)+\beta G]} \cdot G$$

With these results we can also calculate the value of τ where the Hayekian triangle "breaks." Because we have two interest rates (i_g^* and i_p^*) we have two Hayekian triangles. The rate i_g^* defines the slope of the hypotenuse for early stages of production. The rate i_p^* defines the slope for late stages of production. We call the value of τ where both hypotenuses meet τ_B. We can estimate this value from the fact that both levels of consumption are the same (C_B) where the two hypotenuses intersect.

$$(69)\ C_B = (\tau_g - \tau_B) \cdot i_g$$

$$(70)\ C_B = (\tau_p - \tau_B) \cdot i_p$$

$$(71)\ \tau_B = \frac{\tau_p \cdot i_p - \tau_g \cdot i_g}{i_p - i_g} = \alpha$$

$$(72)\ C_B = \frac{(\alpha + \beta)(\bar{Y} - A) - (\beta A + \alpha B)}{\alpha + \beta} < C_g^* < C_p^*$$

4.1 A Numerical Example

As a final application, we offer a numerical example. For brevity, we show only a case for equilibrium and the ABCT case.

Let us calculate first the equilibrium in Garrison's model. Assume that $A=10, B=0, \alpha=0.5, \beta=0.5,\ \bar{Y}=100$. Then, using equations 2 to 6, the equilibrium values are $i^*=10, I^*=5, C^*=95, \tau^*=9.5, APP^*=4.75, H=451.25$.

Assuming now that government increases credit supply by amount $G=2$, using the model in section 4 we can calculate the government and private equilibria and the deviation from Garrison's base scenario equilibrium.

$i_g^*=8$	$i_g^* - i^* = -2$	$i_p^* = 12$	$i_p^* - i^* = 2$
$I_g^*=6$	$I_g^* - I^* = 1$	$I_p^* = 4$	$I_p^* - I^* = -1$
$C_g^*=94$	$C_g^* - C^* = -1$	$C_p^* = 96$	$C_p^* - C^* = 1$
$\tau_g^*=11.75$	$\tau_g^* - \tau^* = 2.25$	$\tau_p^* = 8$	$\tau_p^* - \tau^* = -1.50$
$APP^* = 5.88$	$APP_g^* - APP^* = 1.13$	$APP_p^* = 4$	$APP_p^* - APP^* = -0.75$

With these values we can calculate the change of τ with respect to the increase in credit supply (G): $\frac{\Delta\tau}{\Delta G} = \frac{2.25}{2} = 1.33$. Finally, we can also estimate the point where the Hayekian triangle breaks and the area below the broken triangle:

$$H_{ABCT} = \frac{(\tau_g^* - \tau_B) \cdot C_B}{2} + \frac{\tau_B \cdot (C_p^* - C_B)}{2} + \tau_B \cdot C_B$$

.

Figure 5: Area of the Hayekian Triangle in the ABCT Case
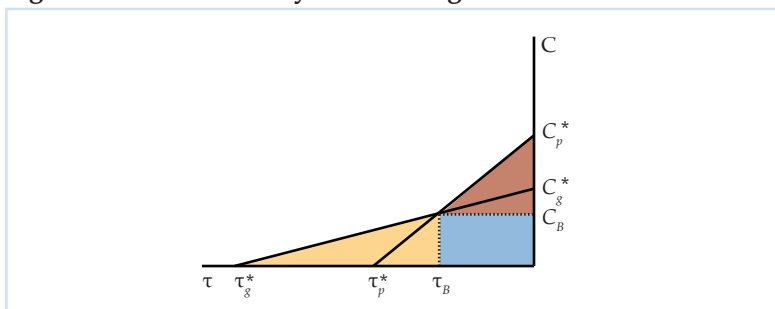
Not surprisingly, this calculation yields a higher value for the area below the hypotenuse than the base case in Garrison's model because private consumption plus investment is outside the PPF by 2, the assumed value of credit expansion; H_{ABCT}=552.75.[12] This is another result that invites to the *over*investment interpretation of the ABCT.

5. CONCLUDING REMARKS

Concurring with Garrison (2001, p. xii), this paper argues that ABCT's graphical model is limited in its ability to develop theoretical extensions to the ABCT and to be subject to empirical falsification. This paper develops a basic mathematical model of the ABCT as an alternative to Garrison's graphical model to avoid some its limitations. In this paper, we also attempt to show how this basic mathematical model is applied and vary when we consider the various applications and extensions that Garrison's (2001) graphical representations cover.

As Garrison's model, the simplicity of our mathematical representation of the ABCT is limited itself in its ability to be empirically tested. There are several possible extensions to the model that can be done to make it more applicable to explain economic crises.

First, two extensions come from applications of the ABCT to the subprime crisis. Cachanosky (2014c) and Young (2012a) apply the

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[12] Because the slopes for demand and supply of loanable funds are the same (in absolute values), consumption and investment both increase each by 1.

ABCT to open economies and add a risk variable. The former does not use Garrison's model, and the latter acknowledges the difficulties of adding financial risk to the graphical version of Garrison's model. A mathematical model would allow adding more variables to the model in order to extend its applicability and help avoid graphical ambiguities. Foreign exchange rates (nominal and real), imports, exports, and risk variables are just a few variables that the ABCT needs to add to be able to fit contemporary business cycles.

Second, there are other possible extensions to the model that could be made to help the model better measure some specific aspects of the ABCT. For example, the model could add a Phillips curve to the model to capture the effects on unemployment during a boom-bust cycle and offer a direct comparison with alternative theories like the Keynesian framework similar to Ravier (2013). The model can measure labor movement across industries by adding a labor market to different stages of production (Garrison, 2001, Chapter 10; Young, 2005). Adding the government sector would allow to analyze the different effects that different ways of financing government spending would have (Ravier and Cachanosky, 2015). Does the government finance the deficit with credit expansion, increase in taxes, domestic debt, or foreign debt?

Instead of looking at the ABCT from a stage-of-production viewpoint, the model could instead incorporate different industries. In Garrison's model, the stages of production are assumed to be well defined and ordered. This assumption fulfills the role of capturing the fact that production takes time and that there is a structure of production that is efficient and avoids shortages or surpluses. But the real world is not divided in similar fashion. Each industry can be thought of as its own triangle and all of them are interconnected providing goods and services to each other (looping). A mathematical version of Garrison's model can add n industries with different *APP* and capture the relative effect on each one of them.

Finally, the model could also incorporate entrepreneurship into its analysis. For example, it could add two entrepreneurs, a *savvy* and a *naïve* one, to show that the ABCT is not built upon representative agents but that relies on heterogeneous entrepreneurs (Cachanosky, 2015a; Callahan and Horwitz, 2010; Evans and Baxendale, 2008). A mathematical framework like the one we present in this

paper opens the opportunity to explore more complex versions of Garrison's model.

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